Is the CAPM valid?
An Empirical Analysis in USA Stock Exchange

Dr. DI Sciorio Fabrizio

Abstract

Quantitative finance was born with the study of the relationship between risk and return. The CAPM is one of the most famous models to estimate the prices of risky assets. It is based on the linear relationship between the excess return and the systematic risk (β). Many studies over the years have analyzed the relationship between risk and return, some of which find weak and inconsistent results. This paper analyzes the CAPM with linear regression highlighting its limits, and proposes a non-linear model for the returns' estimates. The results obtained from the analysis of the NASDAQ-COMPOSITE show greater reliability of the nonlinear model.

Keywords: CAPM, beta, risk, return

***JEL Classification**:* G12, C2.

Introduction

The Capital Asset Pricing Model developed by Sharpe (1964) and Lintner (1965), estimates the return of an asset depending on the investment risk. With this approach we go from an analysis of individual choices (Modern Portfolio Theory) to a decision-making criterion shared among all economic agents. Thanks to the diversification the specific risk of the asset is eliminated α does not appear in the equation. This model is known for the proportionality coefficient called beta, market risk indicator cannot be eliminated through diversification. The hypothesis of the model provide that all investors:

- are rational and risk adverse;
- can land and borrow unlimited amounts under the risk free rate of interest;
- trade withouth transaction or taxation costs;
- have homogeneous expectations.

The classic formulation of the model is:

$$E(R_i)=R_f+\beta_i*[E(R_M)-R_f]$$

$$\text{or } E(R_i)-R_f=\beta_i*[E(R_M)-R_f]\ ;\ \beta=\Sigma\Sigma(R_i-\mu_i)(R_m-\mu_m)/(R_m-\mu_m)^2$$

- where $E(R_i)$ is the expected return on the capital asset;
- R_f is the risk-free rate of interest;
- β is the sensitivity of the expected excess asset returns to the expected exess market returns;
- $[E(R_m)-R_f]$ is know as market premium.

Dr. DI Sciorio Fabrizio, *MD Università degli Studi della Campania "Luigi Vanvitelli" Via Villa Vella, 14 – 81043, Capua (CE)*

Black, Jensen and Scholes (1972), Fama and MacBeth (1973), Gibbons M.R. (1982), M. Murgia (1989), Lamonica (2006) found support for CAPM and their evidence showed a significant positive relationship between realized returns and systematic risk. However, other authors, Banz (1981), Fama and French (1992), Blanco (2012) have found weak or no statistical evidence in support of this relationship. Ross argues that the market portfolio is not empirically observable and usually its proxy uses identified by a market index, which consequently false results obtained. Another hypothesis that is far from reality is the homogeneous expectative hypothesis, that relys on the existence of Homo Oeconomicus.

Black, Scholes and Jensen (1972) introduced the zero beta CAPM. This model provides multiple points of tangency and thus more excellent portfolios going to change the intercept of the LMC. Ross (1978) introduces the APT. In this model the returns of financial assets are influenced by a number of significant regressors and market risk cannot be eliminated by diversification procedures. Just the fact that this additional risk is not diversifiable leads investors to hold assets, require compensation in the form of an excess expected return compared to that provided by non-risky assets. Pettengill, Sundaram, and Mathur (1995) developed a conditional relationship between beta and returns. When the excess return on the market index is positive (negative), we should definitely observe a positive (negative) relationship between beta and returns. In this conditonal study of Pettengill, Sundaram, and Mathur, the empirical results supported the conclusion that there is a positive and statistically significant relationship between beta and realized returns.
The object of this paper is twofold. At the first, I analyze the relationship between beta and returns, using data from the USA Stock Market that might be interesting because it is easy to find information about the companies analyzed. Secondly, I test the nonlinear model(exponential) and I show the difference of the two estimation model.

Data and Methodology

The sample used for analysis is extrapolated from NASDAQ COMPOSITE-index consists of 2584 titles, the selection of the elements of the population took place with causality, which is a necessary condition to be able to adopt the statistical inference methods. With an alpha of 5% and a 95% confidence interval they were extracted 238 titles, through the simple random sampling. We denote by: n the sample size, N the population size (the units of the population are numbered from 1 to N). In the case of sampling without replacement, every unit has probability 1/N to be selected to the first extraction, the remaining have probability 1/(N-1) to the second and so on. Then I divided the sample into homogeneous groups with each other, through a non-hierarchical technical classification. The algorithm used is K-means. The purpose of this step is the division into clusters with elements (titles) internally homogeneous. In mathematical terms, with a within deviance low and a high between deviance. The ex-post returns are calculated: $\sum_{t=1}^{4} \ln(\frac{P_t}{P_{t-1}})$

The model assumptions are:

the terms Rf and [E(Rm)-Rf] are considered constant because the assets are all extracted from the same market. The return of risky activity depends only on the value assumed by the beta. Being two the number of variables involved I have resorted to a simple linear regression model of the type:

$$|Return| = \beta_0 + \beta_1(Risk) + \varepsilon$$

where β_0 and β_1 are respectively the intercept and the gradient of the regression line, and ε is the statistical error associated with the survey. The error is a WN~N $(0, \sigma^2)$.

After estimating, through OLS and Huber's method, the regression model it is necessary to verify the assumptions. For each group I have analyzed the hypothesis: normal distribution, autocorrelation and homoscedasticity. Subsequently, I estimated a nonlinear regression model. In particular, the function is an exponential: $y= a+b*\exp(x)+\varepsilon$ or $Return=a+b*\exp(beta)+\varepsilon$.

Empirical Results

The sample of assets is divided into four groups through K-means

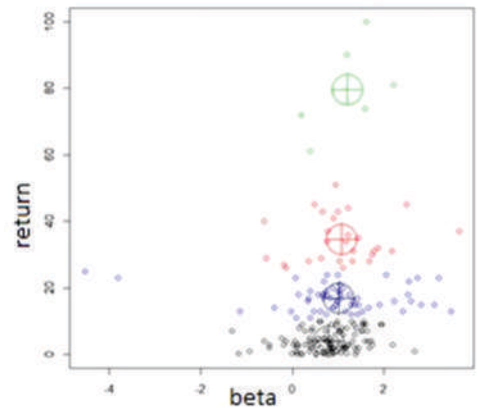

Fig. 1 The figure highlights a classification in four groups (different colors), the cluster elements are arranged evenly around the centroid with η^2=92.1%

The empirical results show the significance of the relationship between return and beta only in a sample of the four analyzed. In particular:

Coefficient	Std Error	T value	Pr(>|t|)
2.8814	0.5832	4.941	3.38e-06***
1.5532	0.5776	2.689	0.00848**

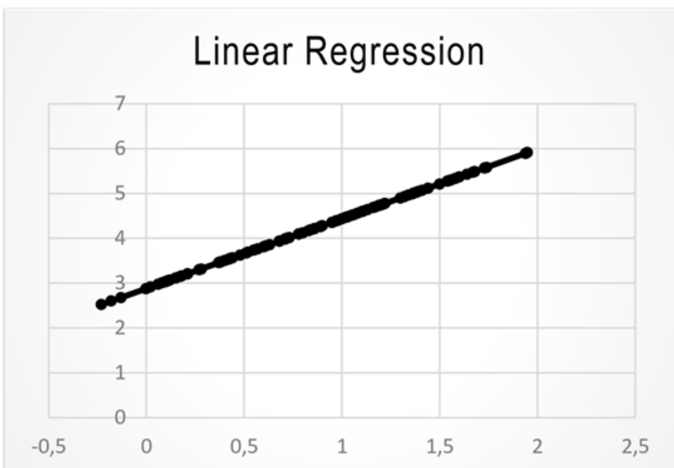

Fig. 2 linear relationship between return and beta

The results seem to confirm the CAPM Theory, due to the significance of the coefficients, but analyzing the R-squared you will notice that the model used does not fit to the observed data.

Test	P-value	R squared
Jarque bera	0.024	0.06154
Bp or ncv	0.456	
DurbinWatson	0.836	

Subsequently, I introduced the nonlinear regression model return=a+b*exp(beta)+ε, where the coefficients are estimated through the NLS method. This method is Nonlinear least square.

Parameters	Std Error	T value	Pr(>|t|)
2.4829	0.6497	3.822	0.000238***
-0.6434	0.2132	-3.012	0.003277**

The model is: Return=2.4829-0.6434*exp(beta)+ε

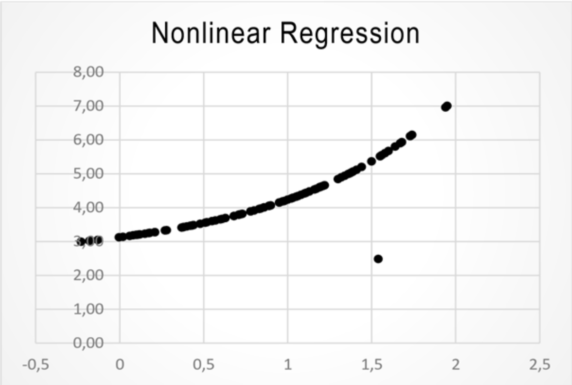

Fig. 3 Nonlinear model

The R-squared =1- RSS/TSS=0.67, confirms the greater reliability on this model than linear regression.
This analysis question the empirical validity of the Capital Asset Pricing Model, as the relationship between return and beta of an assets is not linear.

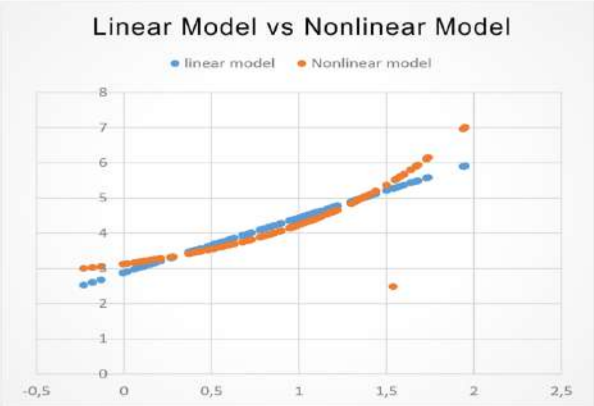

Fig. 4 differences in the estimation of returns between linear and non-linear model

Figure (4) shows the difference in the estimation of returns between the two models. In particular, the linear regression underestimates the returns of the asstes, but in the vicinity of the market portfolio (β=1) they are underestimated. The investor, in his choices, relying on the linear model may underestimate the returns and allocate its resources inaccurately. The results support the hypothesis that there is a relationship between market risk and return, but the relationship is not linear, not confirming the Capital Asset Pricing Model.

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