# MODELING AGGREGATE PRODUCTION PLANNING PROBLEMS AS LINEAR PROGRAMS

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***ABSTRACT***

*Aggregate production planning (APP) seeks to determine production levels for a company's product families over a 12 to 18-month planning horizon. However, the coverage of methods available to create feasible, low-cost aggregate production plans in many production and operations management textbooks does not begin to capture the complexity of aggregate production planning, especially its multiperiod, multiproduct environment. More sophisticated solution procedures such as linear programming receive little or no coverage at all. This paper will explain how to model different APP environments using linear programming. Examples of each environment are included.*

***Keywords:*** *Aggregate Production Planning, (APP), Aggregate Planning, Linear Programming, LP*

## INTRODUCTION

Aggregate production planning (APP) is a medium-range component of a company's overall production planning system. At this planning stage, production levels for product families are determined in order to meet a given demand pattern over the planning horizon (Cox & Stone, 2010).

The minimum cost aggregate production plan may result from any one of three "pure" strategies or a mixed-strategy consisting of a combination of any two, or all three pure strategies Heizer & Render, 2011; Stevenson, 2009). The pure strategies are :

(i) Use inventory and stockouts as a buffer to separate production from demand.

(ii) Change the utilization of the workforce through overtime and undertime to match production with demand.

(iii) Change the size of the workforce through hiring and layoffs to match production with demand.

For students in a first course in operations management, aggregate production planning can be an excellent introduction to the challenges involved in developing a feasible, low-cost production plan.

However, the coverage of methods available to create feasible, low-cost production plans in many production and operations management textbooks does not begin to capture the complexity of aggregate production planning, especially its multiperiod, multiproduct environment. More sophisticated solution procedures such as linear programming receive little or no coverage at all.

Heizer and Render (2011) and Stevenson (2009) discuss why linear programming and other mathematical modeling techniques are not widely used in industry for activities such as aggregate production planning. According to them, mathematical modeling techniques tend not to be accepted by managers as decision making tools because:

(i) Mathematical models are too unrealistic. Managers complain that some modeling assumptions, such as linear costs or a deterministic environment, do not capture the complexity of aggregate production planning and managers will not use them if they are not realistic.

(ii) Mathematical solution techniques are too complicated. Managers do not understand how the techniques work and managers will not use them if they do not understand them.

There are actually two competing model criteria here: the model's level of realism of a planning activity and the level of managerial understanding of how the model works.

Models are abstractions of reality. In model building, there is a trade-off between realism and abstraction but also a trade-off between complexity and simplicity. If the reality of a decision is complex and difficult to understand, a model would be constructed which gives up some of that realism in return for a better understanding of the decision to be made.

Another approach would be to increase managers' understanding of mathematical modeling techniques, specifically how to model various decision environments, so that through experience, more realistic planning models can be constructed. That is our purpose in this paper.

This paper will explain, within the context of the three pure strategies, how to model different APP environments using the mathematical modeling technique of linear programming. The modeled decision problems can then be solved by linear programming software such as LINDO (Schrage, 1997) or a spreadsheet application such as Excel with the Solver add-on .

The next section of the paper reviews the relevant literature. In Section 3, the basic concepts, assumptions, and definitions used throughout the rest of the paper are introduced and a simple production planning problem is created and studied. Section 4 looks more closely at the linkage between inventory and/or stockouts between neighboring time-periods. The discussion will illustrate a variety of options available under Pure Strategy I.

Pure Strategies I and II both involve changing production capacity and are discussed together in Section 5. The formulation and solution of a multiperiod-multiproduct APP problem appears in Section 6. Section 7 concludes the paper with discussion and a suggestion for future research.

## REVIEW OF RELEVANT LITERATURE

Aggregate production planning is an approach to plan for capacity to meet the medium-term demand forecast over a 12 to 18 month period. Aggregate production planning has been a subject of research since the 1950s. Holt, Modigliani and Simon (1955) proposed a Linear Decision Rule (LDR) approach for finding an optimal production and employment schedule, given quadratic cost functions for inventory, labor and overtime costs. In another early work by Bowman (1956) , a specialized linear optimization technique called the Transportation Method was proposed to find the cost minimization solution to production planning. A more generalized approach using Linear Programming (LP) to find an optimal mix of production and employee levels was first proposed by Hanssmann and Hess (1960). The LP approach was extended further by Goodman (1974) to incorporate multiple objective functions by using Goal Programming. Linear programming continues to be a useful tool in modeling and solving large-scale optimization problems, including those involving production planning and scheduling (see for example, Sang-jin and Logendran (1992), Wang and Liang (2005) , and Wu (2010 ).

Others have proposed methods for finding satisfactory, if not optimal, aggregate production planning solutions. Interested readers should see, for example, Bowman (1963), Jones (1967), Taubert (1968) and Lee and Khumawala (1974).

Saad (1982) surveyed and classified then existing aggregate production planning models into two broad categories: descriptive and normative models. Based on Saad's work, Sakalli, (2010) et al. classified traditional models of aggregate production planning into six categories: 1) linear programming, 2) linear decision rule (LDR), 3) transportation, 4) management coefficient approach, 5) search decision rule (SDR), and 6) parametric production planning models. Some of these models have made their way into modern operations management textbooks.

Six current textbooks in production and operations management were reviewed for their coverage of aggregate production planning. All textbooks mentioned the three pure strategies in some form and also the concept of a mixed (or hybrid) strategy.

**Table : Textbook Procedures for Generating an Aggregate Production Plan**

| Author | Trial-and-Error Procedures | Transportation Model |
|---|---|---|
| Heizer and Render (2011) | X | X |
| Stevenson (2009) | X | X |
| Krajewski, et. al.(2010) | X | X |
| Jacobs and Chase (2010) | X | X |
| Reid and Sanders (2010) | X | |
| Swink, et. al.(2011) | X | |

As Table 1 shows, all the textbooks introduced some type of trial-and-error solution procedure. These procedures consist of comparing production and demand levels using period-by-period or cumulative graphs and using spreadsheets to cost out various solutions—usually the three pure strategies. However, both Jacobs and Chase and Reid and Sanders create and cost out a mixed strategy as well.

As for methods that will find the minimum cost aggregate production plan, only Stevenson (2009) and Jacobs and Chase (2010) mention linear programming as a possible candidate. No textbook gives an example using linear programming to model and solve an APP problem.

A modification of the transportation model approach attributed to Bowman (1956) is the only procedure presented that would generate a minimum cost aggregate production plan. To use the transportation model, demand can only be met by using inventory/backorders (Pure Strategy I) and regulartime, overtime, and subcontracting (Pure Strategy II).

The textbook examples are typically 3 or 4 time periods in length for a single product. Options for meeting demand in a time period consist of regulartime and overtime production and subcontracting production to another company. All examples allow the holding of inventory from one period to the next. Only Stevenson introduces the possibility of backordering. Stevenson (2009) is the only author that also suggests that the model can be extended to multiple products.

## THE BASICS

Every linear program is designed to optimize an objective function subject to constraints that define the set of feasible solutions. In the case of aggregate production planning, the goal is to find the plan that minimizes the total relevant cost function. The constraints can be broadly defined as belonging to one of three groups:

1. Inventory "Balancing" Constraints,
2. Simple Upper Bounds,
3. Production Capacity Variation Constraints.

We begin by introducing the following notation (other notation will be introduced as needed throughout the article).

$N$ = total number of products (or product families)

$T$ = number of periods in the planning horizon

$P_{it}$ = number of units of product i produced in period t

$I_{it}$ = ending inventory of product i in period t

$P^{max}_{it}$ = maximum allowable production level of product i in period t

$d_{it}$ = demand for product i in period t

$c_i$ = per unit production cost of product i

$h_i$ = inventory holding cost per unit per period of product i

Our first example is to determine the number of units of each product i to produce in each period t. There is an upper limit on production in each period. The objective is to minimize total production costs plus total inventory holding costs over the planning horizon. The problem can be stated in linear programming form as shown below. The Objective Function (1), to be minimized, and the Constraint Set (2) - (5) will be referred to as the Basic Model. The solution will consist of the values of the variables (production and inventory levels) that satisfy the Constraint Set and result in the minimum value of the Objective Function.

$$\text{Minimize } z = \sum_{i=1}^{N}\sum_{t=1}^{T}(c_i P_{it} + h_i I_{it}) \quad (1)$$

$$\text{subject to: } I_{i,t-1} + P_{it} - I_{it} = d_{it} \quad \text{for all } i,t \quad (2)$$

$$P_{it} \le P_{it}^{\max} \quad \text{for all } i,t \quad (3)$$

$$I_{i0} = 0 \quad \text{for all } i \quad (4)$$

$$P_{it}, I_{it} \ge 0 \quad \text{for all } i,t \quad (5)$$

An assumption we will make is that per unit production costs are relatively constant over time. This results in $\Sigma^{N}_{i=1}\Sigma^{T}_{t=1} c_i P_{it}$ being a constant. Another assumption is that the inventory holding cost per unit per period is constant over time.

The rather simple nature of this model comes about by the fact that we only need to minimize total holding costs. Real production planning problems are usually more complex. Before we get to these more complex (and realistic) models, let's look at this current model more closely. Constraints (2) are our inventory balancing constraints. They express the accountant's inventory balance equation, namely that beginning inventory (which is ending inventory from the previous period) plus production minus ending inventory equals demand (goods sold). This is the basic constraint which can be modified to give all the other kinds of inventory balancing constraints. The upper bounding of production is accomplished by Constraints (3). These constraints are called simple upper bound (SUB) constraints. A typical convention is to assume that the inventory levels at the beginning of the problem are zero (Constraints (4)). If not, these inventory levels should be explicitly stated as constraints. Finally, the nonnegativity conditions are imposed in Constraints (5). The nonnegativity conditions are usually assumed unless there are explicit constraints to the contrary. The nonnegativity of all variables can be assumed in all the following examples.

## INVENTORY BALANCING CONSTRAINTS

Constraints (2) in the Basic Model are used to link production of different periods together by way of ending inventory levels.

A variation of the Basic Model would be to allow backordering. A backorder is the exact opposite of an inventoried item. Inventory occurs when production is greater than demand. Backorders occur when demand is greater than production. Define $B_{it}$ to be the number of units of product i on backorder at the end of period t and $b_i$ to be the cost of having one backorder for product i for one period (we will assume this cost doesn't vary over the planning horizon).

A distinction must be made between inventory level and inventory status. Inventory level is the number of units of an item on hand ($I_{it}$), while inventory status is the difference between inventory level and backorder level ($I_{it}–B_{it}$). Because there are costs associated with holding inventory and backorders, an optimal solution will not have a positive inventory level and a positive backorder level for a product in the same period. The inventory balancing constraint now becomes:

$$\left(I_{i,t-1} - B_{i,t-1}\right) + P_{it} - \left(I_{it} - B_{it}\right) = d_{it} \quad \text{for all } i,t$$

Our new production model with backordering is shown below. Constraints (10) are included to ensure that all demand is met by the end of the planning horizon.

$$\text{Minimize } z = \sum_{i=1}^{N}\sum_{t=1}^{T}(c_i P_{it} + h_i I_{it} + b_i B_{it}) \quad (6)$$

$$\text{subject to: } I_{i,t-1} - B_{i,t-1} + P_{it} - I_{it} + B_{it} = d_{it} \quad \text{for all } i,t \quad (7)$$

$$P_{it} \le P_{it}^{\max} \quad \text{for all } i,t \quad (8)$$

$$I_{i0} = 0 \quad \text{for all } i \quad (9)$$

$$B_{iT} = 0 \quad \text{for all } i \quad (10)$$

A second variation of the Basic Model is to permit lost sales. Lost sales are different from backorders in that they are not carried forward to the next period. Once a unit of demand is "lost", it cannot be recovered in a later period. The inventory balancing constraint which allows lost sales but no backorders is then:

$$I_{i,t-1} + P_{it} - I_{it} + S_{it} = d_{it} \quad \text{for all } i,t$$

where $S_{it}$ is the number of lost sales of product *i* that occur in period *t*. The objective function would contain an appropriate lost sale cost penalty.

A third variation of the Basic Model considers production cost differentials. We will illustrate this by assuming that the cost differentials are due to producing in regulartime and overtime. Let $R_{it}$ = the number of units of product *i* produced in regulartime in period *t* and let $O_{it}$ = the number of units of product *i* produced in overtime in period *t*. Let $R_i$ and $O_i$ be the per unit production costs for regulartime and overtime manufacture, respectively. The inventory balancing constraint (assuming no backorders or lost sales) becomes:

$$I_{i,t-1} + R_{it} + O_{it} - I_{it} = d_{it} \quad \text{for all } i,t$$

The objective function becomes:

$$\text{Minimize } z = \sum_{i=1}^{N}\sum_{t=1}^{T}(r_i R_{it} + o_i O_{it} + h_i I_{it})$$

Additional constraints will be needed to specify the maximum allowable regulartime production and the maximum allowable overtime production. An explanation of how to construct these constraints for various situations is presented in the next section.

## CHANGING CAPACITY

There are several ways to define capacity. For our purposes the two most common are as a number of units or as a number of production hours. If each product always has the same proportion of total capacity or if all products have about the same production rate, capacity expressed in units may be appropriate. For more complex situations, such as when total capacity can be reallocated between products from period to period or products have differing production rates, capacity in terms of a number of production hours is the more appropriate convention. We will define capacity as a number of production hours.

Capacity changes can be classified as either short-term or long-term. Short-term changes to capacity would include the use of overtime and subcontracting. Changing the actual number of workers (and thus the production capacity in a labor intensive operation) would be an example of a long-term capacity change. An assumption of linear programming is that all variables are continuous. Obviously, it is difficult to hire a fraction of a worker. If the number of workers is large, this will usually not be a problem. If the number of workers is small, the linear programming formulation will give a lower bound on the cost of the optimal production plan. The lowest cost feasible production plan could be found by integer programming.

### Short-Term Capacity Changes

Let us go back and consider the situation mentioned at the end of Section 4. Production could be planned to occur either during regulartime or overtime, with appropriate cost penalties (Pure Strategy II). Define $R_t^{\max}$ to be the maximum number of regulartime hours allowed in period *t* and $O_t^{\max}$ to be the maximum number of overtime hours allowed in period *t*. Let $m_i$ be the number of hours it takes to produce one unit of product *i*. Notice that $m_i R_{it}$ and $m_i O_{it}$ will be, respectively, the total number of regulartime hours and overtime hours used to produce product *i* in period *t*. The variation of the Basic Model that permits regulartime and overtime appears below.

$$\text{Minimize } z = \sum_{i=1}^{N}\sum_{t=1}^{T}(r_i R_{it} + o_i O_{it} + h_i I_{it}) \quad (11)$$

$$\text{subject to: } I_{i,t-1} + R_{i,t} + O_{it} - I_{it} = d_{it} \quad \text{for all } i,t \quad (12)$$

$$\sum_{i=1}^{N} m_i R_{it} \le R_t^{\max} \quad \text{for all } t \tag{13}$$

$$\sum_{i=1}^{N} m_i O_{it} \le O_t^{\max} \quad \text{for all } t \tag{14}$$

$$I_{i0} = 0 \quad \text{for all } i \tag{15}$$

Sometimes regulartime wages are guaranteed regardless of worker utilization (*i.e.*, workers are paid for a 40-hour work week, whether or not there is enough work to keep them busy the whole time). In this case regulartime production costs $\Sigma_{i=1}^{N}\Sigma_{t=1}^{T} r_i R_{it}$ are a sunk cost and can be removed from the objective function. The problem then reduces to trading off overtime production costs and inventory holding costs.

**Long-Term Capacity Changes**

Short-term capacity changes assume that maximum allowable regulartime production ($R_t^{\max}$) is constant. When the size of the workforce is allowed to vary (through hiring or layoffs), we have a long-term capacity change. By changing the size of the workforce, maximum regulartime production will now be a variable. Define $H_t$ as the increase to the maximum regulartime hours in period $t$ and $F_t$ as the decrease to the maximum regulartime hours in period $t$. A production capacity balancing constraint is then devised for the production planning model which relates $R_{1-t}^{\max}$ to $R_t^{\max}$, namely,

$$R_{t-1}^{\max} + H_t - F_t - R_t^{\max} = 0 \quad \text{for all } t$$

As appropriate cost penalties for $H_t$ and $F_t$ in the objective function, let $u$ be the cost of increasing capacity by one regulartime hour and let $f$ be the cost of decreasing capacity by one regulartime hour. An optimal solution will not have $H_t$ and $F_t$ both positive in the same period. The variation of the basic model which allows changes in the workforce size is given below.

$$\text{Minimize } z = \sum_{i=1}^{N}\sum_{t=1}^{T}(r_i R_{it} + h_i I_{it}) + \sum_{t=1}^{T}(uH_t + fF_t) \tag{16}$$

$$\text{subject to: } I_{i,t-1} + R_{i,t} - I_{it} = d_{it} \quad \text{for all } i,t \tag{17}$$

$$\sum_{i=1}^{N} m_i R_{it} \le R_t^{\max} \quad \text{for all } t \tag{18}$$

$$R_{t-1}^{\max} + H_t - F_t - R_t^{\max} = 0 \quad \text{for all } t \tag{19}$$

It is possible to have the option of both short-term capacity changes (overtime) and long-term capacity changes (variable workforce size). In this case, both $R_t^{\max}$ and $O_t^{\max}$ will be variables. In practice, overtime is usually limited to a maximum percentage of regulartime hours. For example, if workers regularly work a 40-hour week and can be called on to put in a maximum of an additional 8 hours per week on overtime, the overtime maximum is 20% of the regulartime maximum. Additional constraints will be needed in the production planning model to describe $O_t^{\max}$ as a percentage of $R_t^{\max}$. This is handled by letting $v$ equal the maximum overtime hours allowed as a fraction of regulartime hours and by including the following constraints in the model:

$$vR_t^{\max} - O_t^{\max} = 0 \quad \text{for all } t$$

The linear program of the production planning problem which allows overtime and changes to the workforce size follows. Notice that we do not consider a cost of changing $O_t^{\max}$. One could say that this cost is included in the costs $u$ and $f$ which have to do with changing $R_t^{\max}$. When $R_t^{\max}$ is changed, $O_t^{\max}$ changes as well.

$$\text{Minimize } z = \sum_{i=1}^{N}\sum_{t=1}^{T}(r_i R_{it} + o_i O_{it} + h_i I_{it}) + \sum_{t=1}^{T}(uH_t + fF_t) \tag{20}$$

$$\text{subject to: } I_{i,t-1} + R_{i,t} + O_{it} - I_{it} = d_{it} \quad \text{for all } i,t \tag{21}$$

$$\sum_{i=1}^{N} m_i R_{it} - R_t^{\max} \le 0 \quad \text{for all } t \tag{22}$$

$$\sum_{i=1}^{N} m_i O_{it} - O_t^{\max} \le 0 \quad \text{for all } t \tag{23}$$

$$R_{t-1}^{\max} + H_t - F_t - R_t^{\max} = 0 \quad \text{for all } t \tag{24}$$

$$vR_t^{\max} - O_t^{\max} = 0 \quad \text{for all } t \tag{25}$$

Finally, if regulartime wages are guaranteed, the following Objective Function (26) would replace Objective Function (20):

$$\text{Minimize } z = \sum_{i=1}^{N}\sum_{t=1}^{T}(o_i O_{it} + h_i I_{it}) + \sum_{t=1}^{T}(wR_t^{\max} + uH_t + fF_t) \tag{26}$$

where w is the regulartime wage per hour.

**AN EXAMPLE**

An example is now presented to demonstrate how to formulate a production planning model as a linear program.

**The Problem**

Assume a company produces three product families and must plan monthly production levels for these families over the next six months. In order to satisfy demand, overtime is available when needed and the workforce size can be varied from month to month. Furthermore, workers are guaranteed the full monthly regulartime wage, even if underutilized. This results in a production planning model similar to Objective Function (26) with Constraint Set (21) through (25).

The forecasted demands for each product family are presented in Table 2. Other information relevant to the product families is listed in Table 3.

The regulartime wage is \$12.00 per hour and the overtime wage is \$18.00 per hour. The cost of hiring one worker is \$1200 and the cost of laying off one worker is \$900. Each worker equates to 150 regulartime hours per month (and is guaranteed payment for those 150 hours). Each worker can work up to 37.5 overtime hours per month. The initial number of workers is 35 and the workforce cannot exceed 50 workers in any month.

**Table : 2 Forecasted Monthly Demands**

| | Product Family | | |
|---|---|---|---|
| Month | 1 | 2 | 3 |
| 1 | 4000 | 8000 | 10000 |
| 2 | 5000 | 9000 | 10000 |
| 3 | 6000 | 12000 | 13000 |
| 4 | 6000 | 9000 | 17000 |
| 5 | 4000 | 5000 | 8000 |
| 6 | 1000 | 8000 | 9000 |

**Table: 3 Product Family Information**

| | Product Family | | |
|---|---|---|---|
| | 1 | 2 | 3 |
| Beginning Inventory | 0 | 3000 | 8000 |
| Holding Cost per Unit per Month | \$0.55 | \$0.30 | \$0.35 |
| Production Rate (units per hour) | 2 | 4 | 3 |

The problem is one of ditermining monthly production rule for each product family that will minimize totoal relevant cost.

The total relevant costs are composed of regulartime wages, overtime wages, inventory holding costs, hiring costs, and layoff costs.

The first step is to modify some of the above data into useful model parameters. Some information, such as the holding cost per unit per month and the regulartime wage per hour, needs no modification. The overtime production costs per unit for each product group (o1, o2, o3) are \$9.00, \$4.50, and \$6.00, respectively, found by dividing the \$18.00 overtime wage by the production rates. The per-hour hiring cost (u) is \$8.00 (\$1200 hiring cost per worker divided by 150 regulartime hours per month) and the per hour layoff cost (*f*) is \$6.00 (\$900 divided by 150 hours). The productivity coefficients ($m_1$,$m_2$,$m_3$) will be 0.50, 0.25, and 0.3333, respectively, which are in hours per unit and thus just the reciprocal of the production rates. Beginning regulartime capacity ($R_0^{max}$) is 5,250 hours (150 hours per worker times 35 workers) and regulartime capacity cannot exceed 7,500 hours (150 hours per worker times 50 workers). Finally, the maximum overtime hours allowed as a fraction of regulartime hours ($v$) is 0.25 (37.5 maximum overtime hours divided by 150 regulartime hours).

The resulting linear programming formulation of the production planning example appears below. The linear program will contain 82 variables and 52 constraints. As the number of products (or product families) and length of the planning horizon increases, the number of variables and constraints becomes enormous. If our example had 100 product families and a 24 month planning horizon, there would be 7,397 variables and 2,545 constraints.

$$\text{Minimize } z=\sum_{t=1}^{6}(9O_{1t}+4.5O_{2t}+6O_{3t}+0.55I_{1t}+0.30I_{2t}+0.35I_{3t}+12R_t^{max}+8H_t+6F_t) \quad (27)$$

$$\text{subject to: } I_{i,t-1}+R_{i,t}+O_{it}-I_{it}=d_{it} \quad \text{for all } i,t \quad (28)$$

$$0.50R_{1t}+0.25R_{2t}+0.3333R_{3t}-R_t^{max}=0 \quad \text{for all } t \quad (29)$$

$$0.50O_{1t}+0.25O_{2t}+0.3333O_{3t}-O_t^{max}=0 \quad \text{for all } t \quad (30)$$

$$R_{t-1}^{max}+H_t-F_t-R_t^{max}=0 \quad \text{for all } t \quad (31)$$

$$0.25R_t^{max}-O_t^{max}=0 \quad \text{for all } t \quad (32)$$

$$R_0^{max}=5250 \quad (33)$$

$$R_t^{max}\leq 7500 \quad \text{for all } t \quad (34)$$

$$I_{10}=0 \quad (35)$$

$$I_{20}=3000 \quad (36)$$

$$I_{30}=8000 \quad (37)$$

## The Solution

The optimal linear programming solution to our example was obtained in 90 simplex pivots using LINDO. The minimum value of the Objective Function (27) was \$594,001.75. Again, linear programming assumes that all variables are continuous, and some variables in our solution had fractional values (for example, in the optimal solution $R_{31}$ was 12751.2734 units). If we assume that all variables must be integer and especially that each $R_t^{max}$ must be a multiple of 150 hours (so that the number of workers will be integer if no part-time workers are allowed), a feasible integer solution can be constructed with an objective function value of \$595,331.25. Since the gap between the upper bound and lower bound is small (only \$1329.50 or 0.22%), it is probably not worth the added computational effort to locate the optimal integer solution or verify that our feasible integer solution is optimal.

A summary of the feasible integer solution is given in Table 4 and the cost summary of this solution is presented in Table 5. In Table 4, "RT" denotes the number of units produced in regulartime, "OT" represents the number of units produced in overtime, and "INV" is the ending inventory level.

**Table: 4 Aggregate Production Planning Solution**

| | Product Family | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | 1 | | | 2 | | | 3 | | | |
| Month | RT | OT | INV | RT | OT | INV | RT | OT | INV | Workers |
| 1 | 4000 | 0 | 0 | 5000 | 0 | 0 | 12750 | 0 | 10750 | 50 |
| 2 | 5000 | 0 | 0 | 9000 | 0 | 0 | 8250 | 0 | 9000 | 50 |
| 3 | 3252 | 2748 | 0 | 12000 | 0 | 0 | 8622 | 3 | 4625 | 50 |
| 4 | 2250 | 3750 | 0 | 9000 | 0 | 0 | 12375 | 0 | 0 | 50 |
| 5 | 4000 | 0 | 0 | 5000 | 0 | 0 | 8000 | 0 | 0 | 40 |
| 6 | 1000 | 0 | 0 | 8000 | 0 | 0 | 9000 | 0 | 0 | 37 |

**Table : 5 Aggregate Production Plan Cost**

| | |
|---|---|
| Regulartime Wages | \$498,600.00 |
| Overtime Wages | 58,491.00 |
| Hiring Cost | 18,000.00 |
| Layoff Cost | 11,700.00 |
| Inventory Holding Cost | 8,531.25 |
| Total Cost | \$595,331.25 |

## CONCLUSIONS AND FUTURE RESEARCH

This paper explains how to model different APP environments using the mathematical modeling technique of linear programming. Beginning with a very simple example, the article progresses through increasingly complex formulations. A later section of the article presents and solves a multiperiod-multiproduct APP model. LINDO was used to solve the model and the solution was presented in a series of tables. These examples were designed to help managers easily model a variety of APP environments.

Most operations management textbooks have avoided discussing the application of linear programming for modeling APP problems. Linear programming is deemed to be too complex for managers to understand. However, the authors of this article disagree with this view. Also, for many years, solving linear programming models required specialized software. Now, however, spreadsheet applications are capable of solving linear programming problems-thus eliminating the need for specialized software. Given our view that linear programming is not too complex for managers to understand and given that LP solving tools are now readily available, we believe that introductory operations management textbooks should include LP formulations for APP problems.

Will the availability of LP solving tools be enough to encourage managers to use LP modeling for aggregate production planning? This is a question that needs to be investigated further. Yet, it is widely known that managers are often content with satisficing in their decision-making . That is, they prefer an easy to understand method that may not lead to an optimal decision over a realistic, but more complex, approach. This is evidenced by the coverage of aggregate production planning in many of the currently popular operations management textbooks. These textbooks claim that the easiest of all APP approaches for managers to understand is the graphical method approach. However, the graphical approach compromises when it comes to the realism of the decision environment. The following figure illustrates the tradeoff between model understandability and model realism.

**Model Understanding – Realism Tradeoff**

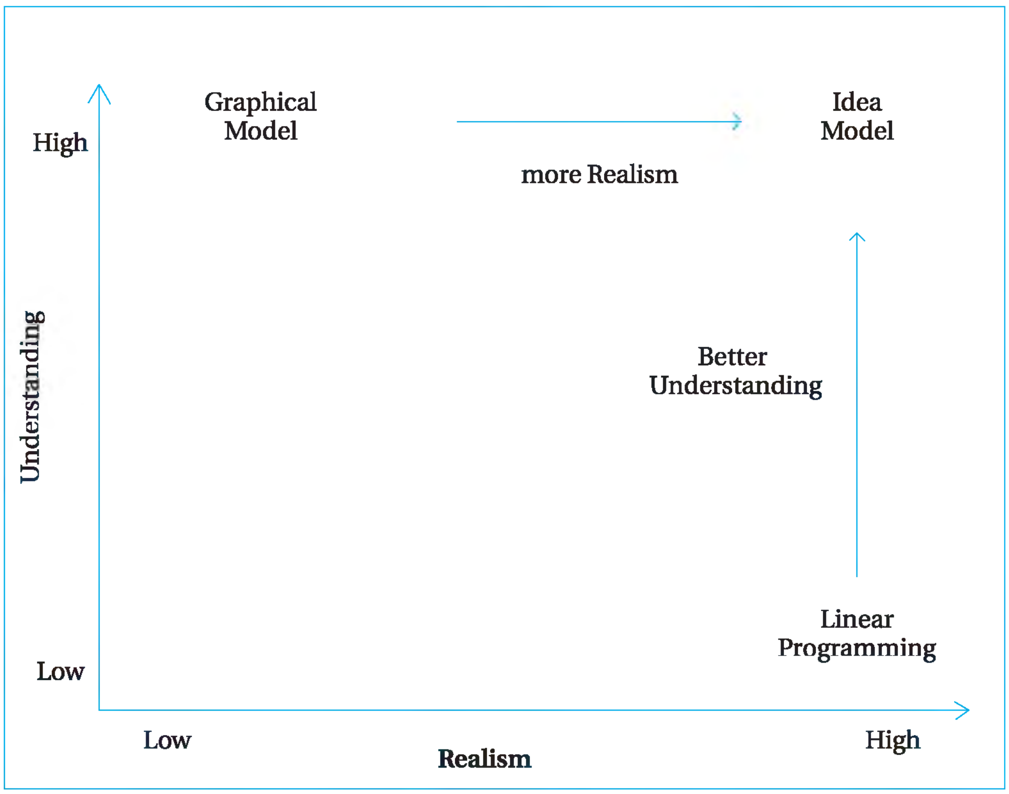


**Figure: 1**

The ideal planning model should have a high level of realism and a high level of understanding by managers. Graphical methods, while easy to understand, lack the ability to effectively model complex planning environments. Linear programming, however, can be adapted to complex decision environments and, as it has been illustrated in the paper, can be made easily understandable to managers. Perhaps a future study can confirm that the linear programming modeling of aggregate production planning is indeed an understandable and practical approach for managers to use.

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1. If the process is capital intensive, long-term capacity changes would be brought about mainly by the addition or removal of manufacturing equipment.
2. The number of product groups is small and the length of the planning horizon is short only so that our solution can be easily illustrated.
3. The optimal integer solution could be found by integer programming. However, this solution's value could not be lower than $594,001.75, thus $594,001.75 is a lower bound. Our feasible integer solution with a value of $595,331.25 is an upper bound on the optimal integer solution. Furthermore, our feasible integer solution might be the optimal integer solution. (The optimal integer solution to this example has an objective function value of $594,731.25.)