SECONDARY SCHOOL STUDENT'S RECURRING PESTILENT ERRORS IN POLYNOMIALS AND ALGEBRAIC WORD PROBLEM SOLVING

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Abstract

This paper reports part of a larger study "Study of Algebraic Misconceptions of Secondary School in Delhi /NCR. The data gathered here leads to the qualitative phase which employs the Case Study Method to gain insights into the thinking process of students which leads to misconceptions in learning of Algebra concepts at the secondary stage. The article presents the findings of a Pilot study which aims to trace the Students Understanding and Misconceptions acquired in Algebra concepts and their implications on solving word problems.

Key Words: *Mathematisation, Cognitive Difficulties, Secondary Mathematics, Word Problems*

INTRODUCTION

Algebra is one of the important strands in Mathematics**.** Understanding and having the proper knowledge and operational skills are very important for students in a secondary school as it facilitates understanding of other topics such as geometry and Trigonometry. It forms the basic language to understand the higher mathematics and hence it determinates whether students will take interest in higher mathematics or not. Mathematics Education is a relatively new area of study. According to the National Curriculum Framework (NCF)" The main goal of Mathematics Education in schools is Mathematisation of child's thinking. The vision of Excellent Mathematical Education is based on the twin premise that all students can learn Mathematics and all students should learn Mathematics". (NCF2005). In the Indian context, Mathematics has been a dreaded subject with a large number of student failures. Cognitive difficulties and Conceptual misunderstandings acquired at the elementary stage hamper the learning and interest in Mathematics. As per NCF 2005 *"Facility with Algebraic manipulation is essential, not*

only for applications of mathematics but also internally in mathematics. Proofs in geometry and trigonometry show the usefulness of algebraic machinery". (Mathematics, n.d.) (Position paper of on Teaching of Mathematics).

Taking note of the low level of mathematics learning, it was decided to reform the curriculum following NCF 2005. The new curriculum is to be child-centered and ensured that learning overall and mathematics learning and teaching is to be an enjoyable experience for the students. ch emphasized child-centered learning. New textbooks for grades 1–12 following the NCF 2005 were brought out by the National Council of Educational Research and Training (NCERT) through a collaborative process involving educators and teachers. The NCERT textbooks in mathematics have undergone several important and noticeable changes in teaching approach, particularly in the primary grades. The presentation of algebra has changed considerably following the NCF 2005. A look at the mathematics textbooks indicates that Algebra has changed from what it used to be i.e. learning to simplify algebraic expressions, using algorithms and word problems far removed from the context of the students. There is an effort to understand the idea of variable, functional relationships and the use of letter numbers in different ways. The attempt is to now understand algebra as a generalization of many of the ideas that are seen as patterns. One of the issues that remain in the new textbooks is the introduction to symbolic algebra in the middle grades, which follows a largely traditional approach focused on symbol manipulation. Algebra is an important part of the secondary curriculum, bringing mathematics to wider sections of the student population, legitimately requires that more thought be given to how algebra can be dealt and developed in a manner that uses students' prior knowledge.

ALGEBRA IN SECONDARY SCHOOL MATHEMATICS ACCORDING TO NCF 2005

A well designed comprehensive curriculum helps to construct and integrate important mathematical ideas to build meaningful conceptual structures. The objectives of the secondary mathematics curriculum are to provide students opportunities to be equipped with important mathematics needed for better educational/professional/social choices. It empowers students to investigate, to understand and make meaning of new situations.

The necessity for solving daily life problems using mathematical language leads to the introduction of 'variables' or 'unknowns' which in turn leading to algebraic expressions, polynomials, linear equations, and their solutions. Thus here for the first time, students get a feel of the Abstract nature of Mathematics. (Teaching Math NCERT 2016). At

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secondary level Mathematics comprises of different topics such as Algebra, Geometry, and Probability, but all these topics stand disconnected. The interconnections among the topics are a must for efficient and resourceful teaching. Students first encounter at the upper primary stage where a slow and difficult transition from arithmetic to Algebra begins. Again treading to Number Patterns after Numbers, seeing the relationship between numbers and forming generalizations lead to the algebraic identities.

STATEMENT OF RESEARCH PROBLEM

There is a desperate need in India especially in Mathematics to explore the reasons so as to why students find mathematics so difficult. This will also provide information about the alarming situation of school dropout for which fear of mathematics and inability to cope up with it is cited as one of the major causes. To address these concerns the researcher framed the following research questions. In the majority of the Indian context, Mathematics is seen as a major hurdle to cross. It is a cause of an alarming number of school dropouts at the secondary level (Pratham Annual Status Report on Education 2016).in an attempt to investigate systematically the possible reasons for the 'fear of Math' algebra featured as one the most difficult to understand and hence the apprehensions about mathematics. Algebra in the upper primary and secondary school curriculum provides a foundation where the higher mathematics concepts rest. An understanding of why and how of the misconceptions acquired at this stage will inform teachers to design their teaching-learning in the classroom for a better understanding of the same.

RESEARCH QUESTIONS

(i) What kind the errors and misconceptions secondary school students make when Working with Polynomials and Linear Equations?

(ii) What are the Implications of these acquired Misconceptions in Problem-solving Word Problems in particular?

THEORETICAL FRAMEWORK

Algebra is typically formally introduced to students in Indian schools in class 6 ie at the onset of the upper primary stage. Students at this stage are most proficient with Arithmetic and the four basic operations. They have had an experience of solving word problems involving the basic four operations of addition, subtraction multiplication and Division .teaching learning of word problems or the story sums even at primary level is challenging for teachers as English is a second language for the majority of the students

poses a problem of comprehension of what is to be done at the onset itself. Slowly and steadily students develop an ease with the procedural fluency to the least and progress their way through the upper primary algebra concepts of Term, Variable, Equations, and problem-solving with variables. Most important they begin to comprehend the abstractness of mathematics at this stage.

Since the secondary School students have already encountered Algebra at the upper primary stage they are aware of the abstract nature of it. Taking cognizance of the fact that a lot of them are struggling with the Arithmetic Algebra Transition Constructivism was taken as the most suitable framework for this study. Constructivism emphasizes that concepts are formed during the learning process when students incorporate new information in their existing schema and modify it. This makes the importance of previous knowledge eminent in gaining new understanding and attaining new knowledge. The constructivist framework asserts that students' efforts to construct knowledge may involve explaining their thinking and reasoning which is an important part of the learning of algebraic concepts that motivated the construction of the research instruments such as the written tests. Thus a collection of previous knowledge, beliefs, preconceptions, and misconceptions help us to look into students making sense of new knowledge.

RESEARCH METHOD AND PROCEDURE

A sequential exploratory design was chosen by the researcher for the study. The Initial Quantitative phase would aid in the selection of the students for detailed interviews in the qualitative part in the later part of the study. At the same time, the results obtained in the qualitative part would explain the why and how of the student's responses to the questions. The purpose of the research was more exploratory than descriptive therefore mixed method research strategy was used by the researcher Seeks to extend the breadth and range of inquiry by using different inquiry method"

POPULATION AND SAMPLING

Systematic random sampling was used to draw one hundred forty-five participants from a population of two hundred and twenty-three students at Kendriya Vidyalayas and Government schools at Delhi/NCR. The schools were selected because the students had the necessary background study of algebra. All participants passed had primary school mathematics. The participants were adolescents in the 15−17 age range. The English language was the medium of instruction for school mathematics learning. The researcher selected class 10 students because it is at secondary school level of learning that students

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are expected to develop a strong foundation for understanding the algebraic concepts that are relevant and necessary for studying mathematics at Senior Secondary level or even pursue it at the higher education level.

A sample is characterized by a group of subjects or people selected from the target population and has the same characteristics. For this study, 75, grade 10 from Kendriya Vidyalayas and 65 students from Government schools in Delhi/NCR were selected using purposive sampling method. Each of the four schools had allowed taking the test for the students who were free in the Zero period when the other students were involved in practice for sports or Co-curricular activities. This group of students made for the quantitative sample of the study. The purposive sampling technique was used to select sixteen students to be interviewed. Four students were selected from each of the four schools. However, since this paper reports only a part of the main study, the results discussed here are for the two Kendriya Vidyalaya in Delhi/NCR.

DATA COLLECTION PROCEDURES

The researcher selected the questions under the required areas of studies using the Central Board of Secondary Education (CBSE) syllabus which is used in all Kendriya Vidyalyas and most of the government schools all over India. The areas selected were Algebraic expressions, Variables, Algebraic equations their implications on word problems. The researcher made a test instrument based on class 8,9 and 10 class curriculum(CBSE) followed in schools across India. The test items were based on two criteria. The first criteria were based on the conceptual understanding of the students, involving the identification of patterns, relationships, and algebraic representation. Some other questions included algebraic Manipulations. Problems involving simplification of equations, Rational Expressions, and word problems. The other type of questions was designed to study the use of the understanding acquired in the above concepts in problem-solving. The word problems had simple easy to understand language and in the context of the students. Questions included justification or reason to be provided by the students, so as to be able to gauge their logic and reasoning. The validity of the content tested was ensured by consulting the same with two experienced math teachers in each school and Teacher Educators teaching Mathematics Pedagogy to prospective teachers. The table illustrates the categorization of questions into the two areas of study. This pilot study is required to reveal any problems in the test instruments and the procedures to be used in the main study. It was also imperative to do the pilot study to make the test instrument reliable and valid.

Concept	Subconcept
Algebraic Expressions	• Simplifying Expressions • Equivalent algebraic Expressions • Comparing algebraic expressions • Forming Algebraic Expressions
Polynomials	• Difference between Polynomial and Algebraic Expression • Value of a Polynomiial at a point • Degree of a Polynomial • Finding Zero of a Polynomial
Equations	• Simultaneous Equations in different formats
Word Problems	• Everyday Language • Mathematical symbolic Language

RELIABILITY

The reliability of the test instrument speaks for its worth and is an important prerequisite as it indicates, how well the test items co-relate with one another. "Measurements are reliable if they reflect the true aspect and not the chance aspect of what is going to be measured" (Gilbert,1989) The researcher used the Split-Half Reliability in the study to get the reliability coefficient.

$$r_{\text{total test}} = \frac{2r_{\text{split half}}}{1 + r_{\text{split half}}}$$

After the first trial of the test, the errors were categorized and a rubric was prepared. This would give an idea of the structure of the content tested and the errors students committed due to misconceptions. The errors under the concept Variables were grouped and the most commonly occurring error was identified. The secondary school teachers at Kendriya Vidyalayas and Government schools helped in this categorization process. With discussions and deliberations on the category of errors, a rubric was created with consensus for each of the four error categories.

VALIDITY

The test instrument was tested for its content Validity. The teachers in the four schools agreed to the content as well as the appropriate difficulty level. They also scrutinized the test paper with regard to the prescribed curriculum of the Central Board of Secondary Education (CBSE). They also had a lot of arguments amongst themselves regarding if a particular concept was relevant to be asked, if it is there in the prescribed NCERT

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textbooks, whether it has been taught in the class or not. The Test instrument was also seen and approved by two senior mathematics teachers and two teacher education experts in the field of Mathematics Education.

The pilot stage was used to modify the test instruments and avoid other possible problems that might show up during the main investigation. Some of the questions were deleted because of either very or very low extreme values. High facility value questions indicated that most of the students could attempt it and the very low facility value indicated very few students even attempted it. Both the cases were not relevant for this study as the misconceptions can be studied only when a student attempts the question and struggle through problem-solving. The researcher then administered the Main test to a group of 140 students in all from the four schools. After evaluating the answer sheets thoroughly and categorizing the errors in the same way as in the Pilot study, four students from each school were selected for Interviews in the final study. Since this report is only a part of the main study the Interview results are not mentioned here.

RESULTS AND DISCUSSION

Student's Misconceptions in Algebraic Expressions

Student's Misconceptions in Algebraic Expressions

Questions	Type of Misconception	Expected Answer	Incorrect Answer	Frequency of Incorrect answer	Percentage
Q7a)	Rational expression error	1	Not possible	06	08
b)	Expression as fraction error	1	2y	07	09
c)	Zero error	0	a	05	06
d)	Factorisation error	(x+y)(x+y)or x^2+y^2+2xy	2(x+y)	06	08
e)	making an equation out of the expression	$\frac{1}{4}$(q+2p-24)	9q+2p-24=0	25	33
f)	Error with bracket opening	$\frac{xa}{b}$	$\frac{xa}{xb}$	05	07
g)	Inappropriate cancellation, x/x taken as 0	$\frac{a+b}{1+d}$	$\frac{a+b}{d}$, $1+\frac{b}{d}$	08	11
Q6	Equivalence error in Rational Expression	$\frac{x}{2x}-\frac{3}{2x}$	$\frac{x-3}{1}$=2x	20	27

Q8	Inappropriate cancellation due to lack of understanding of distributive law	$\frac{A(C+B)}{BC}$	AC +A/C	07	09
Q12	Giving values to x and comparing the magnitude of denominator instead of the whole fraction.	$\frac{1}{N}$	I/N is a Natural Number. It is inversely proportional	06	08
Q13 a)	Converting Expression to Equation error	(x+y+z)	(x+y+z)=0	31	41
b)	Lack of closure property for algebra letters	7+4x	Not Possible	32	43
c)	Like terms error	2x+2c+5p	x^2 +2c+5p	05	07

Students made mistakes when they multiplied algebraic fractional expressions. For instance, for the question Simplify (ax/), the major error observed was that the students multiplied both the numerator and the denominator of the fraction by the letter to get *ax/bx*. Sometimes they did not take cognizance of the denominator. It happens when; it appears that there is no denominator. They have difficulties in realizing that a single letter can represent by an algebraic fraction with 1 as the denominator. Students think that both numerator and denominator of the fraction should be multiplied by the letter. Errors occurred when previous learning interfered in new learning. Above table shows the most prevalent errors among the students. These were adding unlike terms and formulating and subsequent solving of irrelevant equations. The error of adding unlike terms, that the students failed to realize that an algebraic expression 7+4*x* can be the final answer cannot be simplified. Students at times do the mathematical procedures without understanding the implicit meaning of the procedures and why they are required to be done. Finally, students' difficulties with algebraic expressions were manifested in the solution attempts to the task. Simplify *ax+xb/x+xd*. Common incorrect answers were *a+bd* or *a+bd* that emerged from processes in which in students correctly factorised out *x* in both numerator and denominator but failed to divide denominator and numerator by *x* leading to incomplete answers such *as x(a+b)/x(1+d) and x(a+b)÷x(1+d)*.In other solutions, they just crossed out x.

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Student's Misconceptions in Polynomials

	Type of Misconception	Expected Answer	Incorrect Answer	Frequency of Incorrect answer	Percentage
Q2 Which of the given Expression polynomial and which is not. Give reasons.	Difference between polynomial and Algebraic Expression	a) is polynomial	Various incorrect answers	25	33
Q1 If $P(x)=5-4x+12x^2$ Find a) P(3) b) P(-2)	Meaning of Zero of Polynomial Value of Polynomial at a point	a) 101 b) 61	Different incorrect answers	31 27	41 36
Q3 Write the Degree of the Polynomials a), b), c) Q4) Give an Example of Binomial of Degree 27.	Degree of Polynomial	a) 1 b) 0 c) 7	. Different incorrect answers	08 07	11 09

Polynomials as a concept are very important for student's Mathematical understanding. If students are proficient in the operations such as addition subtraction Multiplication of Polynomials, they get an important tool in mathematics that facilitates their meaningful understanding of the other topics in high school algebra in particular and mathematics in general. The above table shows most students displayed a very shallow or no understanding of the meaning of Polynomial. They were only aware of the procedural knowledge and rules of computation. For example, most of them understood and could write polynomial with a given degree but lacked an understanding of the relationship of the degree of a polynomial with the number of solutions and Zero of the polynomial. This incomplete understanding gives rise to a number of misconceptions about Polynomials and difficulties in understanding other related concepts in mathematics.

Student's Misconceptions in solving Equations

	Type of Misconception	Expected Answer	Incorrect Answer	Frequency of Incorrect answer	Percentage
Q16	Procedural	X=-1	X=0	12	16
Q17	Wrong operations in substitution method	(0,4)	..	19	25
Q18	Added the equations most of the where subtraction was required. Ignored the denominator in 2y/3 and added 2y/3 and 2y	$(4,\frac{-1}{2})$	Several wrongs due to incorrect transposing and sign errors.	41	55
Q10	Did not understand the structure of the subtraction statement. "Subtract "was taken as an order to minus	10−2b	2b−10	07	09
Q20	Was trying to solve the equation and not understand the balancing role of "=" symbol	Balancing equation with values of m and n	m−n=2	29	39

Student's solution attempts to the task use the elimination to solve the simultaneous equations x+y=4; y=2x+4. The students' answers revealed procedural errors occurred when students were in the process eliminating the unknown from the two linear equations. The students added the two equations to eliminate x stead of subtracting. This misconception is due to an incomplete understanding of simplifying integers and manipulating signs. They failed to realize they could still obtain the solutions by adding or subtracting two equations.

Pupil's Misconceptions in Word Problems

Questions	Type of Misconception	Expected Answer	Incorrect Answer	Frequency of Incorrect answer	Percentage
Q14	*Language errors. The direct translation of Keywords to symbols. *Number of times mathematical operations occurred interferes with forming expressions	x=5 here the emphasis was also on *the reasoning and *formulating an equation after reading the word problem. *Solving the equation errors were observed Minus sign errors occurred	15 and various incorrect answers	09	12

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Q15	Linguistic errors, The relational word error. Use of two Variables C for coffee and D for Dosa.	4d=5c	4c=5d	25	33
Q19	Inability to understand relational words and hence fails to represent relationship mathematically.	G=B+3	B=G+3	28	37

The questions asked on the word problems wherein the students had to attempt problem-solving were in accordance with the requirements set by NCF (2005) and the national curriculum framework of Teacher Education NCFTE (2009). The problem-solving process in the math classroom should view students as active participants and not just recipients of knowledge.

Problem-solving situations provide an excellent opportunity for students to construct their knowledge and reject misconception acquired earlier if any. This is also an opportunity to apply classroom knowledge to the real world and their immediate context.

Here the word problems are treated as a subject on their own. It will also an indication of how the understanding acquired in the concepts of algebraic expressions, linear equations and variables facilitate or interfere with the ability to solve word problems. The problem-solving process here involves the following routes.

- Make sense of word problem
- To represent the mental diagram of the problem.
- To identify the given and the unknowns in the word problems
- To retrieve the required known knowledge for the specific word problem.
- To establish a mathematical relationship between the unknowns and knowns
- To solve the mathematical equation.
- To translate the mathematical variable to the original unknown.

A word problem is presented as a story problem in normal language, it has to be read and understood clearly first and the given and unknowns identified very clearly. Therefore using an unambiguous language is as important that situating the word problem in the context of the students. Then the structure of the word problem has to be identified and the relation between the given and unknown sorted. Then follows the process of finding the solution which can use different methods as shown in the figure above. It came to

light after detailed interviews that the students were taught how to attempt and solve the story sums in different steps could successfully arrive at the solution.

CONCLUSION

It appears from the discussions of the students' responses that student's ability to do word problems is affected by the language. Indian classroom is multicultural and students from different cultural background, a different mother-tongue are sharing space in the classroom. The teaching-learning in the classroom is mostly in English, which is not their first language. Therefore the inability to comprehend a word problem at the first place demotivates them from further attempting to solve it before changing into mathematical equation or expression. For effective algebra learning to happen in especially in Indian Classroom the two-level Language barriers have to definitely take care of by the teachers in the upper primary level itself. An elaborate discussion of mathematical symbols and signs, when and where to be used is imperative as it makes the students comfortable to converse in Mathematical language. This also adds to the confidence level and instills liking for the subject which is the necessary requirement for any kind of child-centered teaching learning to take place. A basic principle behind constructivist teaching-learning is to understand that students' responses to the activity are meaningful to them, no matter how wrong it seems to the others. It is very important for the teacher to interpret the students thinking and rationale behind the response and correct it in agreement with the student. So one should not look at student's errors as roadblocks, but a stepping stone to make the concept clear. Errors provide an opportunity for the teacher to look into students thinking and plan their teaching-learning to suit student's needs. This approach to errors and mistakes committed by the students will definitely replace the drill and endless practice of questions and replace them with more meaningful learning.

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